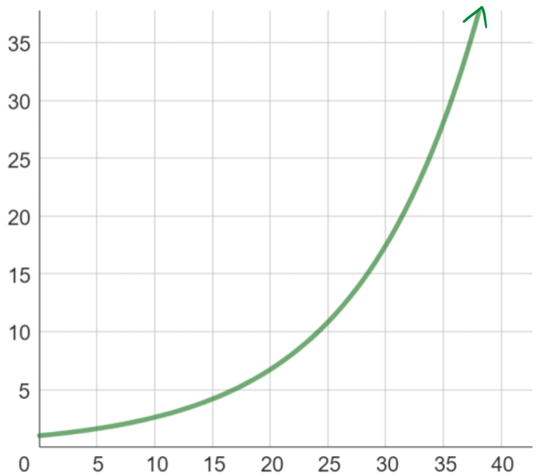
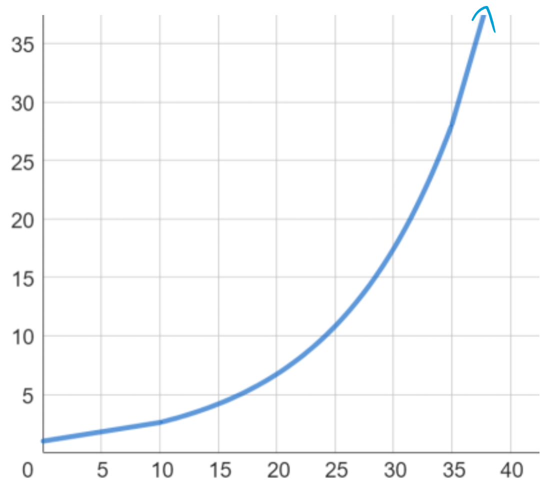
Log Functions Day 7:
(2.15) Semi-log Plots

Homework: Semi-log Plots

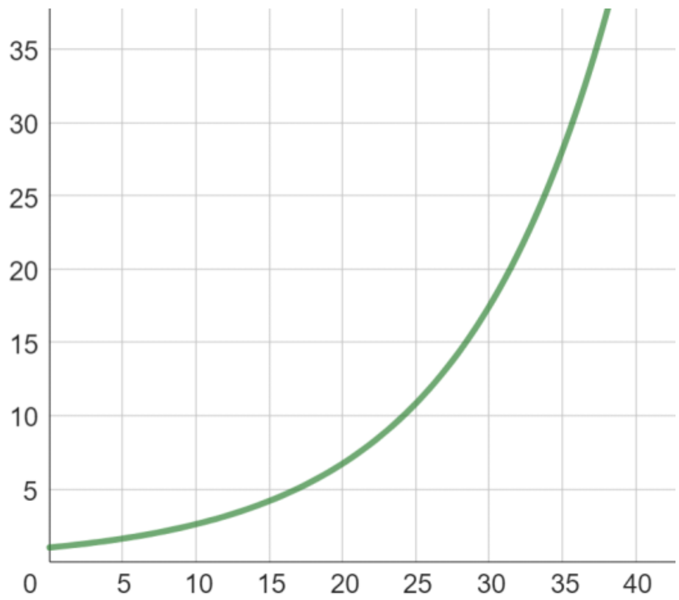
Quiz Next Class:
- Solving Log Equations
- Solving Log Inequalities
- Solving Exponential Equations
- Solving Exponential Inequalities
- Finding inverses of log/exponential functions

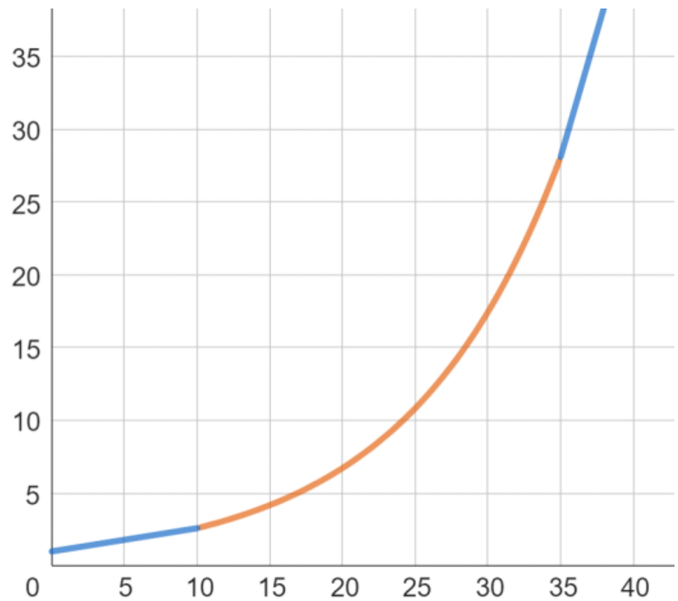
What is the difference between the two graphs?





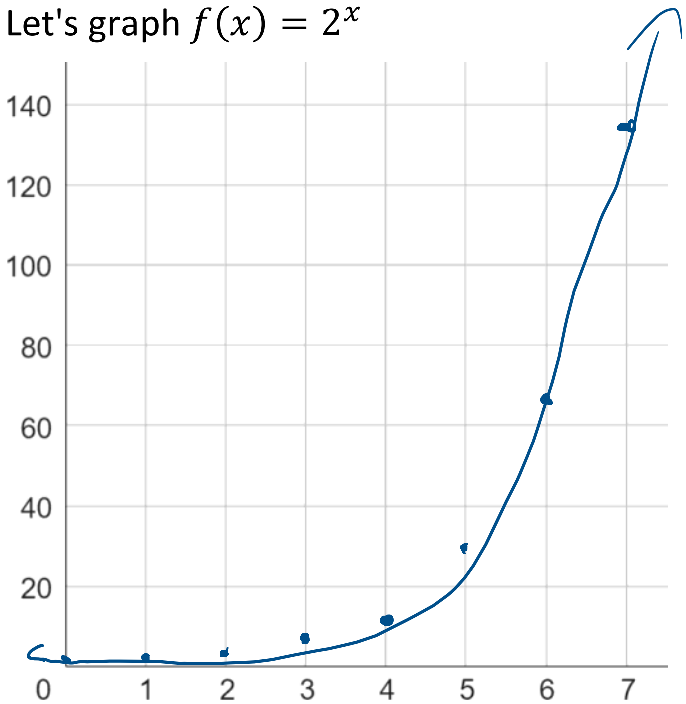
The one on the right was linear at the edges! We couldn't even tell! Why couldn't we tell?

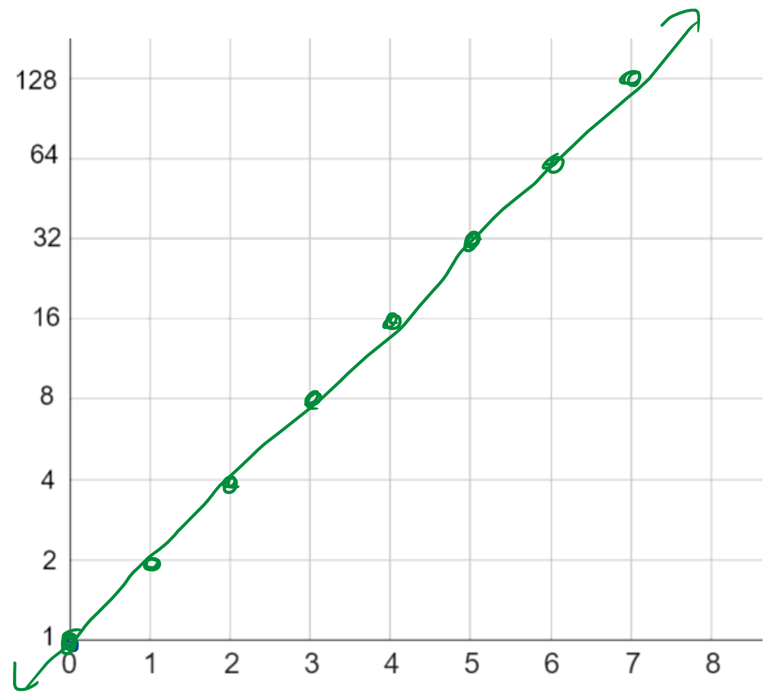




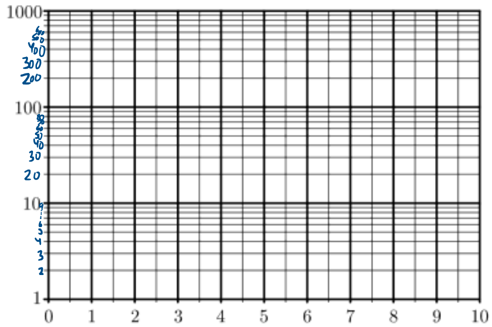
Graphing with Linear Scale vs Graphing with Logarithmic Scale

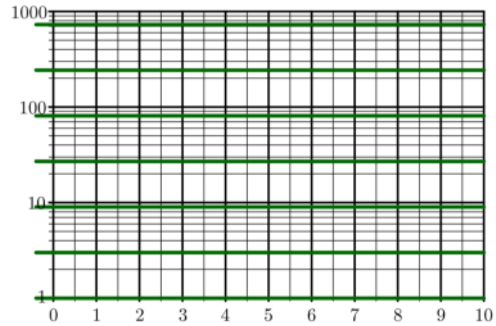
Let's graph $f(x) = 2^x$



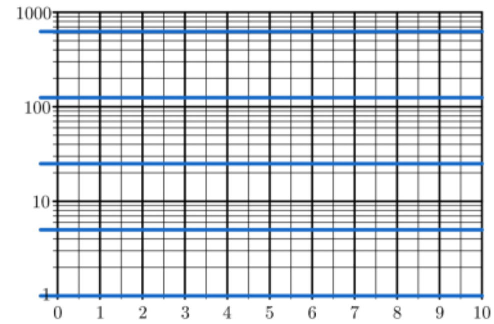


On a semi-log plot, equally spaced vertical axis values are proportional.



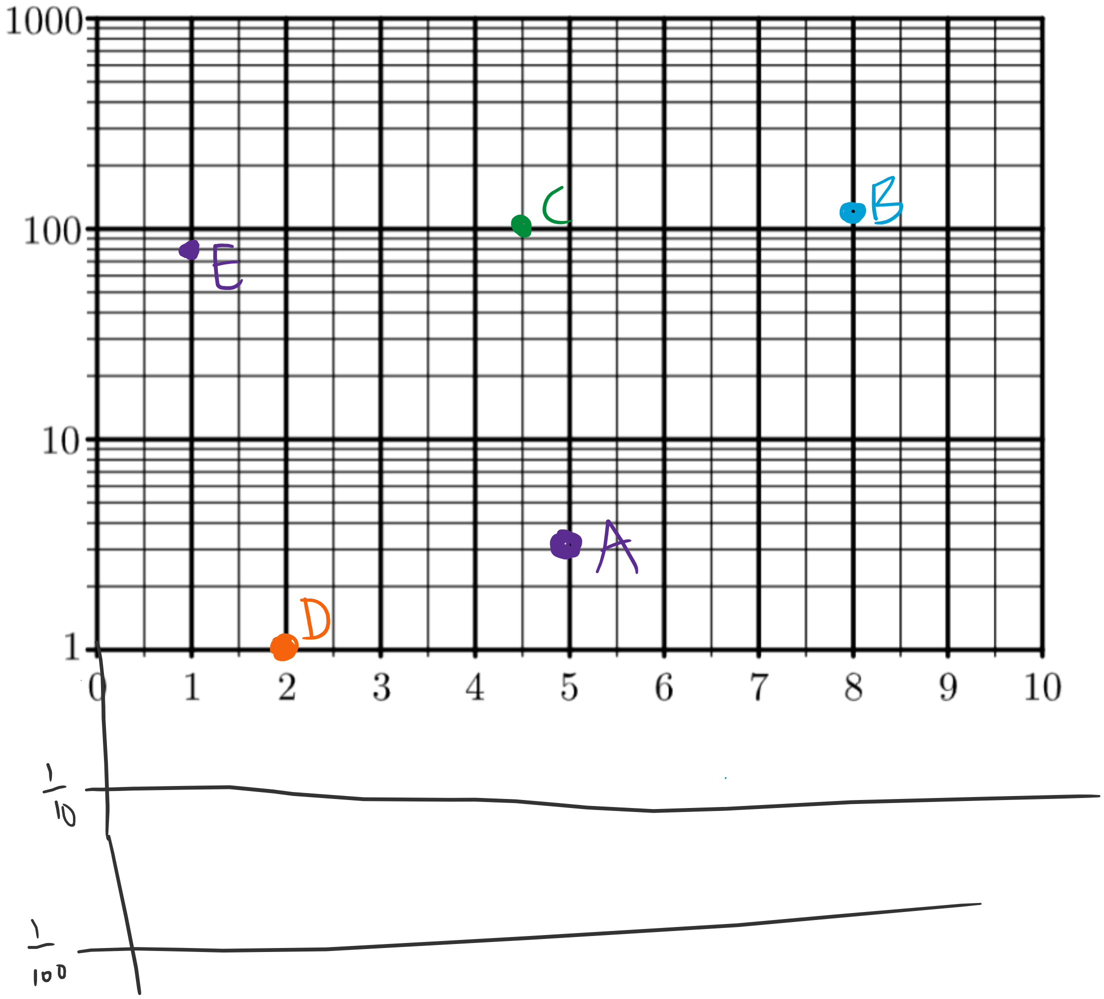


Green lines at 1, 3, 9, 27, 81, 243, 729



Blue lines at 1, 5, 25, 125, 625

Let's practice plotting points



Plot:

A) (5,3)

B) (8,120)

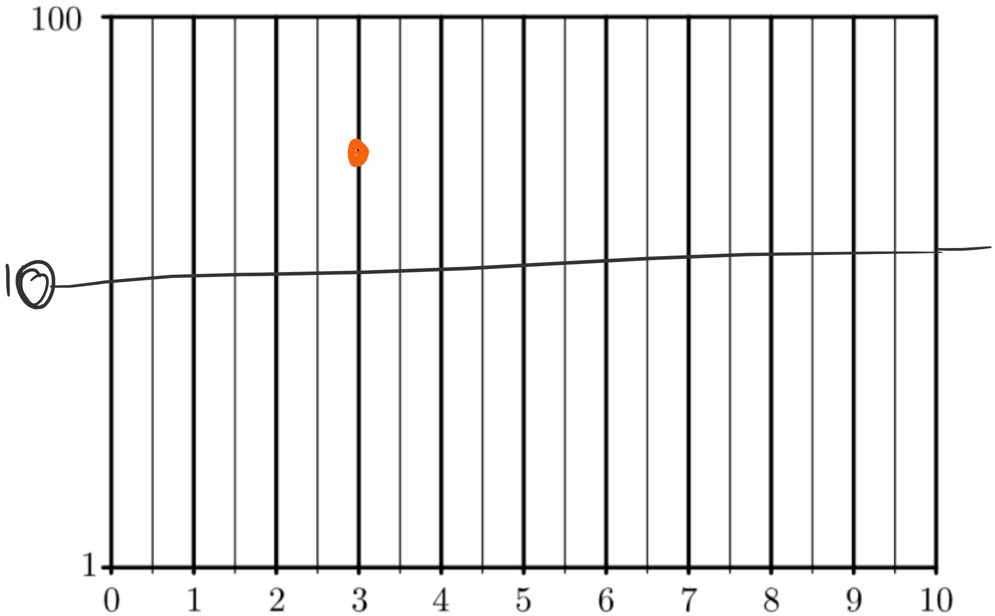
C) (4.5, 100)

D) (2,1)

E) (1,80)

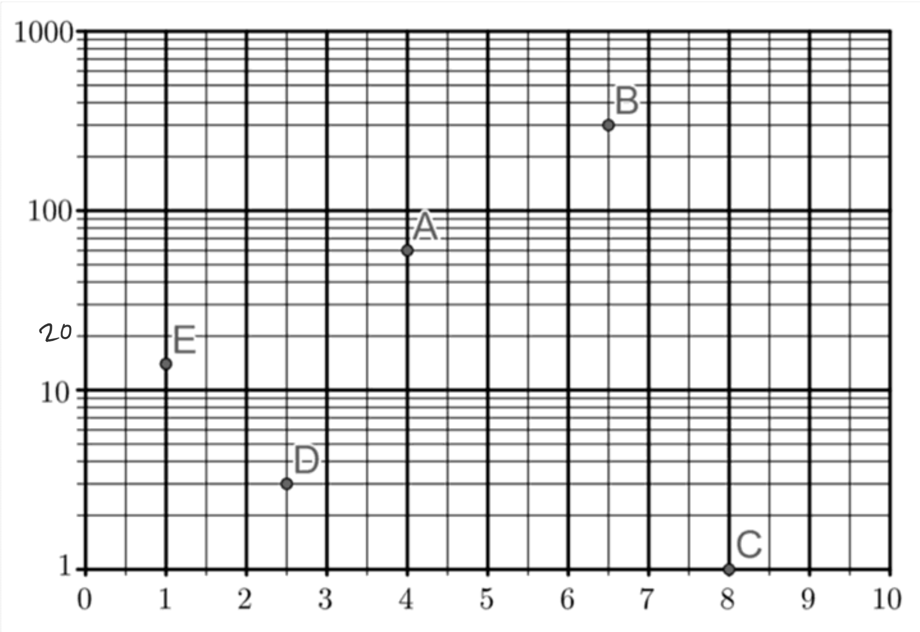
F) (9, −1)

Let's practice plotting points



Plot the point $(3,50)$

Let's practice reading points



Find the coordinates for each point.

$A =$ (4, 60)

$B =$ (6.5, 300)

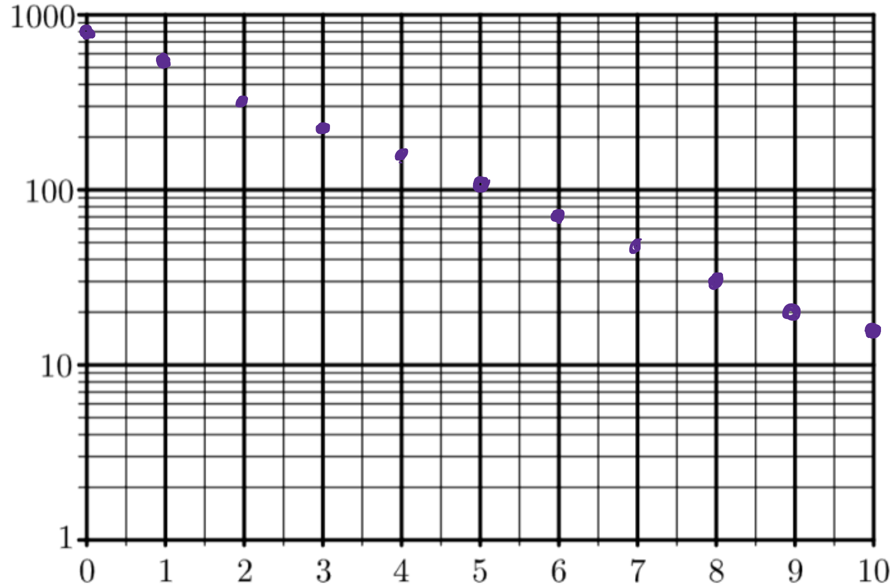
$C =$ (8, 1)

$D =$ (2.5, 3)

$E =$ (1, 14)

Useful things about semi-log plots

- Useful thing #1: You can see the difference in output values for small or large input values to exponential functions.
- Useful thing #2: Exponential functions appear linear. So, if data appears linear on a semi-log plot, you know the data could be modeled well by an exponential function!



Plot:

A) (0,800)

B) (1,550)

C) (2, 360)

D) (3,240)

E) (4,160)

F) (5,106)

G) (6,70)

H) (7,50)

I) (8,30)

J) (9,20)

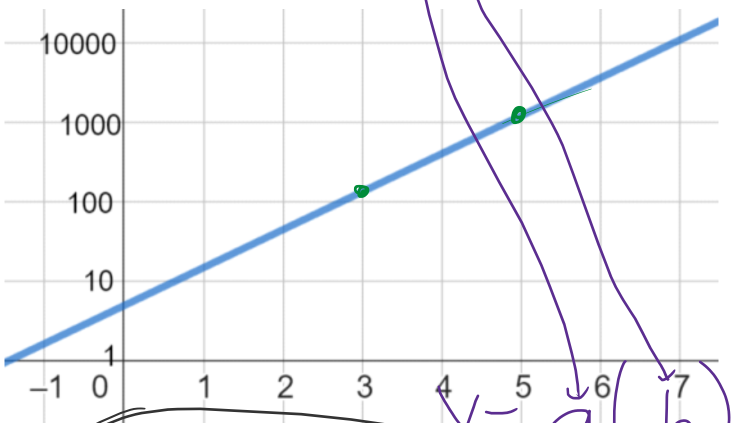
K) (10,15)

There are only about 4 things you need to know about semi-log plots,
so at the risk of repeating myself…

- If you have data that follows an exponential model, the semi-log plot will appear linear
- If the semi-log plot appears linear, you have data that will follow an exponential model.

A little less talking and a little more math…

Check out this graph of $f(x) = 5(3)^x$ plotted with a logarithmic vertical scale.



I wonder what the slope and y-intercept of this line are…

slope looks like 0.5

initial value looks like 0.75

$y = a(b)^x$

Slope $= \log_n b$

Slope $= \log_{10}(3)$

Slope $= 0.477$

Initial Value (y-intercept) $= \log_n a$

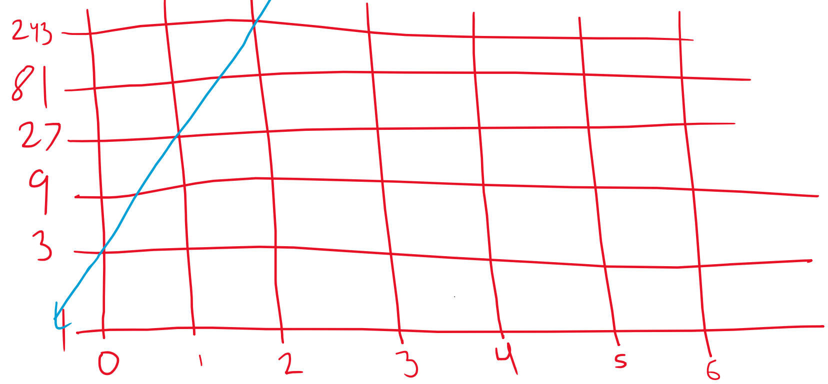
Initial value $= \log_{10}(5)$

Initial Value $= 0.699$

There are only about 4 things you need to know about semi-log plots,
so at the risk of repeating myself…

- If you have data that follows an exponential model, the semi-log plot will appear linear.
- If the semi-log plot appears linear, you have data that will follow an exponential model.
- For $y = ab^x$, the corresponding linear model has slope $\log_n b$ (where $n > 0$ is whatever is the ratio between your equally spaced vertical axis tick marks)
- For $y = ab^x$, the corresponding linear model has initial value $\log_n a$ (where $n > 0$ is whatever is the ratio between your equally spaced vertical axis tick marks)

A data set that appears exponential is modeled by the function f. When the data are represented using a semi-log plot, where the vertical axis is scaled using a base 3 logarithm, a linear pattern is apparent. The linear model corresponding with the semi-log plot has equation $y = 2x + 1$. Find an expression for f in the form $f(x) = ab^x$



$f(x) = 3(9)^x$

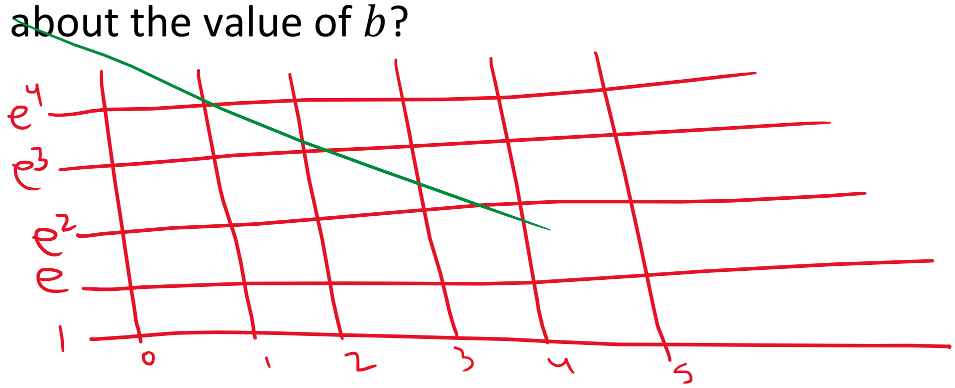
Slope $= \log_n b$. initial value $= \log_n a$

$3^{(2)} = 3^{(\log_3 b)}$ $3^{(1)} = 3^{(\log_3 a)}$

$9 = b$ $3 = a$

A data set that appears exponential is modeled by the function f given by $f(x) = ab^x$. When the data are represented use a semi-log plot, where the vertical-axis is scaled use a natural log, a linear pattern is apparent.

If the slope of the linear pattern is negative, what does that imply about the value of b?



exponential decay

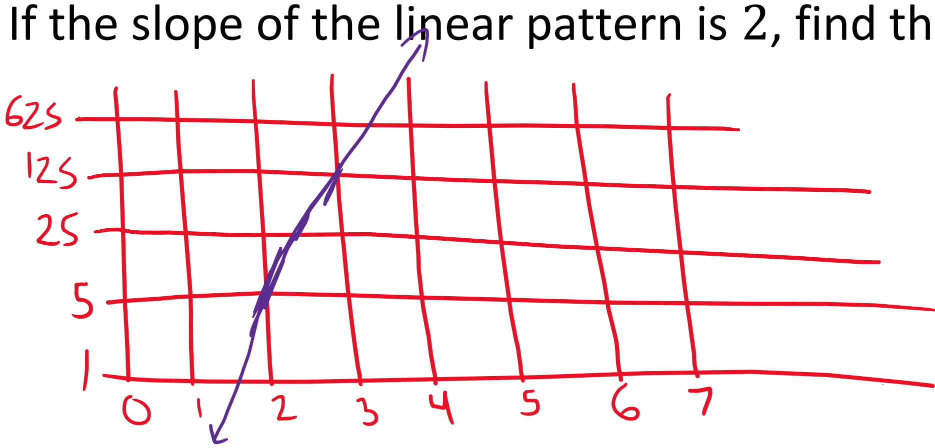
$0 < b < 1$

A data set that appears exponential is modeled by the function f given by $f(x) = ab^x$. When the data are represented use a semi-log plot, where the vertical-axis is scaled use a natural log, a linear pattern is apparent.

If the initial value of the linear model is positive, what does that imply about the value of a?

A data set that appears exponential is modeled by the function f given by $f(x) = ab^x$. When the data are represented use a semi-log plot, where the vertical-axis is scaled use a log base 5, a linear pattern is apparent.

If the slope of the linear pattern is 2, find the value of b



$$\text{Slope} = \log_n b$$

$$5^{2} = 5^{\log_5 b}$$

$$5^2 = b$$

$$25 = b$$

A data set that appears exponential is modeled by the function f given by $f(x) = ab^x$. When the data are represented use a semi-log plot, where the vertical-axis is scaled use a log base 5, a linear pattern is apparent.

If the initial value of the linear model is $\frac{1}{2}$, find the value of a

Day 5 Solving Log Equations HW

Day 5
Solving ...

Solving Log Equations (2.13) Homework

For questions #1-10, solve the equation. Remember to check for extraneous solutions.

1. $3\log_2(x-3)+1=13$

2. $\frac{\log(3-x)+5}{2}=2$

3. $\ln(x+4)+\ln(x-3)=\ln(18)$

4. $\log_3(x+3)-\log_3(x-3)=1$

5. $\log_3(x-2)-\log_3(2x+1)=0$

6. $\ln(x)+3\ln(2)=\ln(x^2+12)$

7. $2\log_6(x+1) = \log_6(x+7)$

9. $2\log_4(x-2) = \log_4(3) + \log_4(8-x)$

8. $\ln(2x+5) - 1 = \ln(5)$

10. $\log_3(x-1) = 3 - \log_3(2x+1)$

11. Consider the functions f and g given by $f(x) = \log_5(3x+2)$ and $g(x) = \log_5(x-1) + \log_5(x+6)$. In the xy-plane, what are all x-coordinates of the points of intersection of the graphs of f and g?

12. Let $f(x) = \log_2(x-1)$ and $g(x) = \log_2(x+2) - 1$. Find all values of x where the graphs of f and g intersect.

$4 = (\log_2 x)^2 - 3(\log_2 x)$

13. The function g is given by $g(x) = (\log_2 x)^2 - 3(\log_2 x)$. Find all input values in the domain of g that yield an output value of 4.

$4 = A^2 - 3A$

$0 = A^2 - 3A - 4$

$0 = (A-4)(A+1)$

$A = 4 \qquad A = -1$

$(\log_2 x) = (4) \qquad (\log x) = (-1)$

$A=4$ $A=-1$

$2^{(\log_2 x)} = 2^{(4)}$ $2^{(\log_2 x)} = 2^{(-1)}$

$x=16$ $x=\frac{1}{2}$

Log Equations HW Answers

1. $x = 19$
2. $x = \frac{29}{10}$
3. $x = 5$
4. $x = 6$
5. No Solutions
6. $x = 2, 6$
7. $x = 2$
8. $x = \frac{5e-5}{2}$
9. $x = 5$
10. $x = 4$
11. $x = 2$
12. $x = 4$
13. $x = 16$ and $x=\frac{1}{2}$

Day 6 Exponential and Log Inequalities HW

Day 6 Exponential and Log Inequalities HW

Exponential and Log Inequalities (2.13) Homework

1. Let $f(x) = 2 + \log_3\left(\frac{x}{2} - 4\right)$. Find all x-values for which $f(x) \le 3$.

2. Let $g(x) = 20 - 4e^x$. Find all values of x for which $g(x) < 12$.

$20-4e^x < 12$

$20-4\boxed{e^x} = 12$

$-4e^x = -8$

$e^x = 2 \rightarrow x = \ln 2$

$20-4e^0 < 12$

$20-4(1) < 12$

$20-4e^{10} < 12$

$20-4(2.\ldots)^{10} < 12$

$(\ln 2, \infty)$

3. Let $f(x) = \log_2(x+2)$ and $g(x) = 5 - \log_2(10-x)$. For what input values is the output of $f(x)$ less than the output of $g(x)$?

4. Let $f(x) = \log_4(-x+8) + \log_4(x+2)$ and $g(x) = \log_4(4x+1)$.
   a. Find all values of x for which $f(x) \ge g(x)$.
   b. Find all values of x for which $f(x) < g(x)$

$-x+8>0$

$8>x$

$x+2>0$

$x>-2$

$4x+1>0$

$4x>-1$

$x>-\frac{1}{4}$

$\log_4(-x+8) + \log_4(x+2) \ge \log_4(4x+1)$

$\log_4(-x+8) + \log_4(x+2) = \log_4(4x+1)$

$4^{\left(\log_4((-x+8)(x+2))\right)} = 4^{\left(\log_4(4x+1)\right)}$

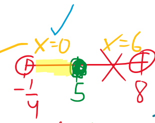
$(-x+8)(x+2) = 4x+1$

$-x^2+6x+16 = 4x+1$

$0 = x^2-2x-15$

$0 = (x-5)(x+3)$

$x=5 \quad x=-3$



$\left(-\frac{1}{4}, 5\right]$

$\log_4 8 + \log_4 2 \ge \log_4 1$

$\frac{3}{2} + \frac{1}{2} \ge 0$

$\log_4(2) + \log_4(8) \ge \log_4 33$

$\frac{1}{2} + \frac{3}{2} \ge \log_4 25$

$\frac{1}{2} + \frac{3}{2} \ge \log_4 25$

$2 \ge$ 2.something

$\log_4 32 = \frac{5}{2} = 2.5$

$\log_4 16 = 2$

5. Let $f(x) = 5^{x-1} - 4$. Find all input values for which the output of f is negative.

6. Let $f(x) = \log_2(x+2)$ and $g(x) = \log_2(x-3) + 1$. Find all x-values for which $f(x) \le g(x)$.

7. Let $f(x) = \log_3(11-x) - \log_3(x+1)$. Find all values of x for which $f(x) \le 1$.

8. Let $f(x) = 10 - \log_5\left(\frac{x}{7}\right)$. Find $f^{-1}(x)$.

9. Let $g(x) = -2 + 5\log_3(4x)$. Find a function h that satisfies $(g \circ h)(x) = (h \circ g)(x)$ for all x-values.

$\frac{x-8}{4} > 0$

$x-8>0$

$x>8$

10. Let $f(x) = 3\log_2\left(\frac{x-8}{4}\right)$.
   a. ~~Find the domain~~ and range of f.

   D: $(8,\infty)$ Range: $(-\infty,\infty)$

   b. Find the domain and range of f^{-1}.

   D: $(-\infty,\infty)$ R: $(8,\infty)$

   c. Find $f(12)$. $= 3\log_2\left(\frac{12-8}{4}\right) = 3\log_2(1) = 3(0) = 0$

   d. Find $f^{-1}(12)$. $=72$

   $12 = 3\log_2\left(\frac{x-8}{4}\right)$

   $2^{(4)} = 2^{\left(\log_2\left(\frac{x-8}{4}\right)\right)}$

   $16 = \frac{x-8}{4}$

   $64 = x-8$

   $72 = x$

   e. Find $f^{-1}(x)$.

$72 \rightarrow \boxed{f} \rightarrow 12$

$12 \rightarrow \boxed{f^{-1}} \rightarrow 72$

$x = 3\log_2\left(\frac{y-8}{4}\right)$

$2^{\left(\frac{x}{3}\right)} = 2^{\left(\log_2\left(\frac{y-8}{4}\right)\right)}$

$2^{x/3} = \frac{y-8}{4}$

$4(2)^{x/3} = y-8$

$4(2)^{\frac{x}{3}} + 8 = y$

Exponential and Log Inequalities (2.13) HW Answers

1. $(8,14]$
2. $(\ln 2, \infty)$
3. $(-2,2) \cup (6,10)$
4. a. $\left(-\frac{1}{4}, 5\right]$
   b. $(5,8)$
5. $(-\infty, 1+\log_5 4)$
6. $[8,\infty)$
7. $[2,11)$
8. $f^{-1}(x) = 7(5)^{10-x}$
9. $h(x) = g^{-1}(x) = \frac{1}{4}(3)^{\frac{1}{5}x+\frac{2}{5}}$
10.
   a. Domain of f: $(8,\infty)$ Range of f: $(-\infty,\infty)$
   b. Domain of f^{-1}: $(-\infty,\infty)$ Range of f^{-1}: $(8,\infty)$
   c. $f(12) = 0$
   d. $f^{-1}(12) = 72$
   e. $f^{-1}(x) = 4(2)^{\frac{x}{3}} + 8$

Exponential and Log Inequalities (2.13) HW Answers

1. $(8,14]$
2. $(\ln 2, \infty)$
3. $(-2,2) \cup (6,10)$
4. a. $\left(-\frac{1}{4}, 5\right]$
   b. $(5,8)$
5. $(-\infty, 1+\log_5 4)$
6. $[8, \infty)$
7. $[2,11)$
8. $f^{-1}(x) = 7(5)^{10-x}$
9. $h(x) = g^{-1}(x) = \frac{1}{4}(3)^{[illegible]}$
10.
   a. Domain of f: $(8, \infty)$ Range of f: $(-\infty, \infty)$
   b. Domain of f^{-1}: $(-\infty, \infty)$ Range of f^{-1}: $(8, \infty)$
   c. $f(12) = 0$
   d. $f^{-1}(12) = 72$
   e. $f^{-1}(x) = 4(2)^{\frac{x}{3}} + 8$

$16 = \frac{x-8}{4}$

$64 = x-8$

$72 = x$

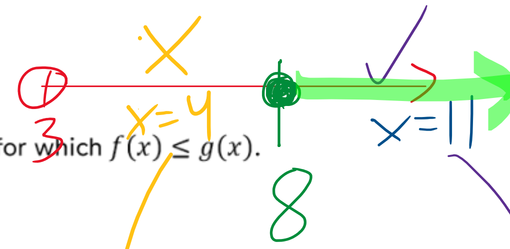
$2^{\left(\frac{x}{3}\right)} = 2^{\left(\log_2\left(\frac{y-8}{4}\right)\right)}$

$2^{x/3} = \frac{y-8}{4}$

$4(2)^{x/3} = y-8$

$4(2)^{\frac{x}{3}} + 8 = y$

CBHS AP Precalculus 2024/25 – Logarithmic Functions, Day 6 HW



6. Let $f(x) = \log_2(x+2)$ and $g(x) = \log_2(x-3) + 1$. Find all x-values for which $f(x) \le g(x)$.

$\log_2(x+2) \le \log_2(x-3)+1$

$x+2>0$

$x>-2$

$x-3>0$

$x>3$

$\log_2 6 \le \log_2 1 + 1$

2.something $\le 0+1$

$\log_2(x+2) = \log_2(x-3)+1$

$\log_2(x+2) - \log_2(x-3) = 1$

$2^{\left(\log_2\left(\frac{x+2}{x-3}\right)\right)} = 2^{(1)}$

$\frac{x+2}{x-3} = 2$

$x+2 = 2(x-3)$

$x+2 = 2x-6$

$8 = x$

$\log_2 13 \le \log_2 8 + 1$

3.something $\le 3+1$

$[8, \infty)$

Let $f(x) = 5^{x-1} - 4$. Find all input values for which the output of f is negative.

$5^{x-1} - 4 < 0$

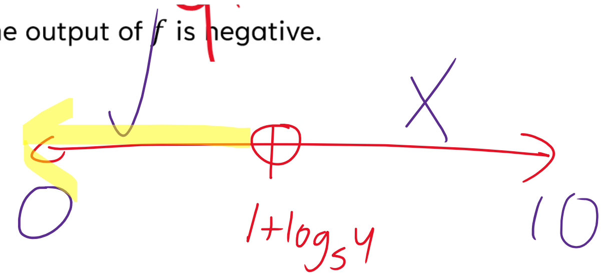
$5^{x-1} - 4 = 0$

$5^{x-1} = 4$

$\log_5(5^{x-1}) = \log_5(4)$

$x - 1 = \log_5 4$

$x = 1 + \log_5 4$



$5^{-1} - 4 < 0$

$\frac{1}{5} - 4 < 0$

$5^{9} - 4 < 0$

Massive Number $- 4 < 0$

Let $f(x) = \log_2(x+2)$ and $g(x) = 5 - \log_2(10-x)$. For what input values is the output of $f(x)$ less than the output of $g(x)$?

$\log_2(x+2) < 5 - \log_2(10-x)$

$\log_2(x+2) = 5 - \log_2(10-x)$

$x+2>0$

$x>-2$

$10-x>0$

$-x>-10$

$x<10$

$\log_2(x+2) + \log_2(10-x) = 5$

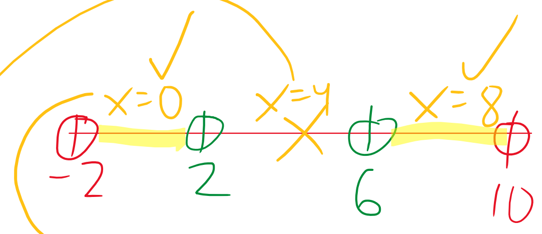
$2^{\left(\log_2((x+2)(10-x))\right)} = 2^{(5)}$

$(x+2)(10-x) = 32$

$-x^2+8x+20 = 32$

$0 = x^2-8x+12$

$0 = (x-6)(x-2)$



$\log_2 2 < 5 - \log_2 10$

$1 < 5 - 3.\text{something}$

$\log_2 6 < 5 - \log_2 6$

$\log_2 6 + \log_2 6 < 5$

$\log_2 36 < 5$

$5.\text{something} < 5$